Complementary Root Locus Revisited

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Abstract—**In this paper, a new finding related to the well-known root locus method that is covered in the introductory control systems books is presented. It is shown that some of the complementary root locus rules and properties are not valid for systems with loop transfer functions that are not strictly proper. New definitions for root locus branches have been presented which divide them into *branches passing through infinity* and *branches ending at or starting from infinity*. New formulations for calculating the number of branches passing through infinity, point of intersection of the asymptotes on the real axis, and angles of these asymptotes with the real axis have been introduced. It has been shown this type of system with the order of n will have at least one and at most n branches which will pass through infinity. The realization and stability of these systems have been investigated, and their gain plots have been presented. The new finding can be used by educators to complement their lecture materials of the root locus method. By using problems similar to examples presented in the paper, analytical understanding of the students in a classical control systems course can be tested.**

Index Terms—**Control systems, gain, root locus, stability, transfer functions.**

I. INTRODUCTION

THE ROOT locus plot was introduced for the design of feedback control systems by Evans in 1948 [1] and since then has become a standard and commonly used tool in control system education and practice. Almost all the introductory textbooks for control system analysis and design used in most of the undergraduate engineering disciplines have devoted one chapter to the root locus construction rules and properties [1]–[7]. It is a set of theorems and techniques that calculates the locations of the closed-loop poles in the s-plane as a changing parameter (gain) in the open-loop transfer function varies over some defined range. The plot of positive gain is known as root locus (RL), and the locus of negative gain is referred to as complementary root locus (CRL).

The root locus plot has been the center of numerous educational and research activities [8]–[29]. Undergraduate control systems textbooks still give emphasis to root locus plotting [5]–[7], and research papers present computational methods [14], [15], computer implementations [16], [17], and new approaches [18]–[22] for the root locus method. The method is an essential design and analysis tool in most of the control system software [16], [17]. This technique has been a valuable tool in characterizing response and stability of linear systems and has been utilized extensively for designs in control engineering practices [23]–[29].

Although use of RL is more common than CRL, it has been shown that the latter can be very valuable in design studies via the principle of continuation [22], can be utilized for changing time constants [8], and can be used when large loop gain is required in inner loops [6].

In this paper, first a couple of examples are presented to show that some of the construction rules for the root locus plot are not valid for a specific class of feedback control systems with a negative gain. Then, new definitions are provided for root locus branches and new formulations are given for calculating the number of branches passing through infinity, point of intersection of the asymptotes on the real axis, and angles of these asymptotes with the real axis for the root locus plot.

The examples provide a foundation for those who are teaching introductory control systems course to design their own exercises to test the student understanding of the analytical concepts of the root locus technique.

Consider the following closed-loop transfer function with negative gain:

$$T(s) = \frac{C(s)}{R(s)} = \frac{KG(s)}{1 - KG(s)H(s)}. \tag{1}$$

The complementary root locus is obtained by changing the value of K from zero to infinity.

From the root locus construction rules [1]–[7], it is known when the number of poles and zeros of the loop transfer function is equal; there is no root at infinity. Therefore, there are no asymptotes in the locus when the loop transfer function is not strictly proper.

This conclusion is true for all different types of negative feedback control systems, but it is not valid for positive feedback systems with loop transfer functions that are not strictly proper.

To see this, use (1) to obtain the complementary root locus for the following two systems whose loop transfer functions are not strictly proper:

1)

$$G(s)H(s) = \frac{2s^4 + 5s^3 + 6s^2 + 8s + 12}{3s^4 + 8s^3 + 9s^2 + 12s - 16}.$$

The root locus of this system is shown in Fig. 1. Since $m = n = 4$ there should not be any roots at infinity; however, as shown in Fig. 1, there is a root at infinity in the locus.

2)

$$G(s)H(s) = \frac{2s^4 + 4s^3 + 6s^2 + 8s + 20}{3s^4 + 6s^3 + 9s^2 + 12s - 10}.$$
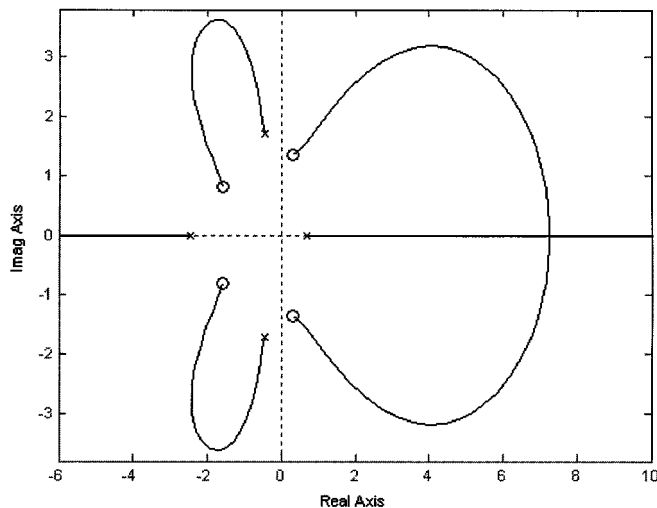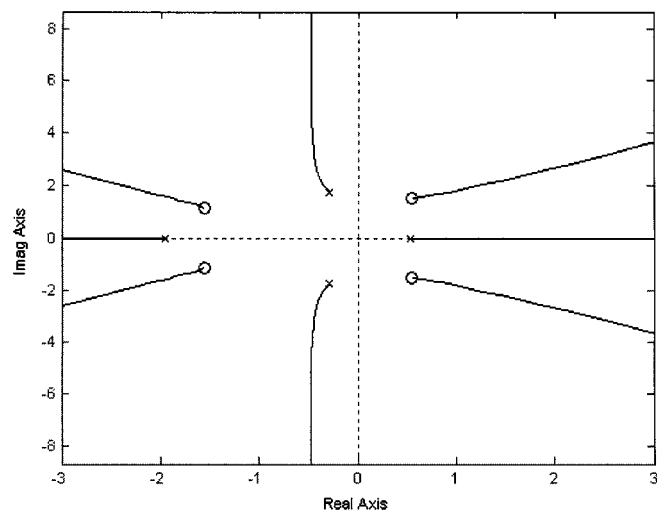
Fig. 1.   Root locus of example 1.

Fig. 2.   Root locus of example 2.

The root locus of this system is shown in Fig. 2. The order of the numerator and the denominator are the same in this system, but as shown in Fig. 2, there are four roots at infinity in this locus.

With these examples, it is obvious that the root locus construction rules have to be modified. For this purpose, the branches of locus are divided into two groups. The first group contains branches ending at or starting from infinity, and the second group consists of branches passing through infinity.

Definition 1: The branch ending at or starting from infinity either starts from a finite pole and ends at an imaginary zero at infinity or starts from an imaginary pole at infinity and ends at a finite zero. In this branch, the value of K at infinity is either zero or infinity.

Definition 2: The branch passing through infinity starts from a finite point, goes through infinity, and ends at another finite location. In this branch, the value of K at infinity is a finite value.

The branch ending at or starting from infinity is the same branch that is familiar and has been introduced in various control books and papers [1]–[7]. However, root locus of positive

feedback systems whose loop transfer function has the same number of poles and zeros consists of branches that are passing through infinity. This type of branch has not been introduced in any books and publications until now. In the next section, formulations will be introduced for calculating the number of branches passing through infinity, point of intersection of the asymptotes on the real axis, and angles of these asymptotes with positive direction of the real axis.

In this paper, the branches with a point at infinity will be called "infinite branches."

II. Properties of Branches Passing through Infinity

Use the following loop transfer function, which is not strictly proper, in (1):

$$G(s)H(s) = \frac{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}{b_n s^n + b_{n-1} s^{n-1} + \cdots + b_1 s + b_0}$$

$$a_n \neq 0, \quad b_n \neq 0, \quad \forall (a_i, b_i) \in R. \quad (2)$$

The root locus of this system has properties defined by the following theorems.

Theorem 1—Gain of System for Poles at Infinity: The gain of these systems approaches a constant and finite value for infinite poles.

As a result, infinite branches in the locus of these systems are branches passing through infinity.

Proof 1: The characteristic equation of the system defined in (2) is

$$\Delta(s) = 1 - KG(s)H(s) = 0. \quad (3)$$

From (3) K is calculated as follows:

$$K = \frac{1}{G(s)H(s)}. \quad (4)$$

When a pole of the system goes to infinity, (4) is written as

$$\lim_{s \to \infty} K = \lim_{s \to \infty} \frac{1}{G(s)H(s)}$$
$$= \lim_{s \to \infty} \frac{b_n s^n + b_{n-1} s^{n-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}. \quad (5)$$

The value of K for infinite poles will be called critical gain, K_c, and is obtained from (5) as $s \to \infty$

$$K_c = \frac{b_n}{a_n}. \quad (6)$$

So infinite branches of these systems are branches passing through infinity.

It is concluded from the proof of this theorem that positive feedback systems with loop transfer functions that are not strictly proper do not have any branches ending at or starting from infinity, but they have one or more branches passing through infinity.

Theorem 2—Locus on the Real Axis: In these systems, both ends of the real axis are located on the root locus.

Proof 2: In positive feedback systems with loop transfer functions that are not strictly proper, the total number of poles and zeros of loop transfer function is $2n$ and it is even.

Therefore, as it was shown in [5]–[7], the right side of the last pole/zero on the right and the left side of the last pole/zero on the left of the real axis are parts of the root locus plot. As a result, there is always at least one branch passing through infinity in the root locus of these systems.

Theorem 3—The Number of Branches Passing through Infinity in Root Locus: The number of branches passing through infinity is related to the corresponding coefficients of the same power of s in numerator and denominator of the loop transfer function and is equal to the maximum index number j of the ratios in the following orderly relation:

$$K_c = \left(\frac{b_n}{a_n}\right)_1 = \left(\frac{b_{n-1}}{a_{n-1}}\right)_2 = \left(\frac{b_{n-2}}{a_{n-2}}\right)_3 = \cdots = \left(\frac{b_{n+1-j}}{a_{n+1-j}}\right)_j. \tag{7}$$

Proof 3: For a specific value of K, the order of the characteristic equation may reduce to $(n-m)$ which means the order of the characteristic equation reduces m degrees. This value of K corresponds to n points in the locus that m of them are located at infinity. If the value of K_c is substituted in the characteristic equation given by (3), the following relation will be obtained:

$$\Delta_c(s) = 1 - K_c G(s)H(s) = 0. \tag{8}$$

Combining (6) and (8) gives

$$\Delta_c(s) = 1 - \frac{b_n}{a_n} G(s)H(s) = 0. \tag{9}$$

Substituting (2) into (9) results in

$$1 - \frac{b_n}{a_n}\left(\frac{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}{b_n s^n + b_{n-1}s^{n-1} + \cdots + b_1 s + b_0}\right) = 0,$$

$$b_n s^n + b_{n-1}s^{n-1} + \cdots + b_1 s + b_0$$
$$- \frac{b_n}{a_n}\left(a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0\right) = 0$$

or

$$\left(b_{n-1} - \frac{b_n a_{n-1}}{a_n}\right)s^{n-1} + \left(b_{n-2} - \frac{b_n a_{n-2}}{a_n}\right)s^{n-2} + \cdots$$
$$+ \left(b_1 - \frac{b_n a_1}{a_n}\right)s + \left(b_0 - \frac{b_n a_0}{a_n}\right) = 0. \tag{10}$$

It is concluded from (10), that for $i = 0$, the coefficient of $s^{n-i} = s^n$ always becomes zero, then the root locus of systems indicated with (2) has a branch passing through infinity. This result confirms theorem 2.

Also for $i = 1$, if the coefficient of $s^{n-i} = s^{n-1}$ becomes zero, then

$$b_{n-1} - \frac{b_n a_{n-1}}{a_n} = 0$$

or

$$\frac{b_n}{a_n} = \frac{b_{n-1}}{a_{n-1}} = K_c. \tag{11}$$

Therefore, there are two branches passing through infinity in root locus.

If (11) is satisfied and for $i = 2$, the coefficient of $s^{n-i} = s^{n-2}$ becomes zero, then

$$\left(b_{n-2} - \frac{b_n a_{n-2}}{a_n}\right) = 0$$

or

$$\frac{b_n}{a_n} = \frac{b_{n-1}}{a_{n-1}} = \frac{b_{n-2}}{a_{n-2}} = K_c. \tag{12}$$

So, there are three branches passing through infinity in root locus.

Thus, if coefficients of s^{n-i} where $i = 0, 1, 2, \ldots, (m-1)$ are all zero, then

$$\frac{b_n}{a_n} = \frac{b_{n-1}}{a_{n-1}} = \cdots = \frac{b_{n-(m-1)}}{a_{n-(m-1)}} = K_c. \tag{13}$$

Therefore, the root locus of this system contains m branches passing through infinity. With the result of this theorem, the locus of these systems always contains at least a branch passing through infinity, and the order of their characteristic equation for K_c is at most $(n-1)$.

Theorem 4—The Asymptotes of Branches Passing through Infinity: If m branches passing through infinity exist in the root locus of a system defined by (2), then the root locus of this system contains $2m$ asymptotes, m asymptotes for that part of the branches going to infinity, and m asymptotes for that part of the branches coming from infinity. All of these $2m$ asymptotes are centered at a point on the real axis given by

$$\sigma = \frac{\dfrac{a_{n-m-1}K_c - b_{n-m-1}}{a_{n-m}K_c - b_{n-m}} - \dfrac{a_{n-1}K_c}{b_n}}{m}. \tag{14}$$

The angles of the asymptotes of the branches passing through infinity that are going to infinity with respect to the real-axis are

$$\theta_i = \frac{2i\pi}{m} \quad \text{where} \quad i = 0, 1, 2, \ldots, m-1. \tag{15}$$

The angles of the asymptotes of the branches passing through infinity that are coming from infinity with respect to the real axis are

$$\theta_i = \frac{(2i+1)\pi}{m} \quad \text{where} \quad i = 0, 1, 2, \ldots, m-1. \tag{16}$$

Proof 4: The system given by (2) contains m passing through the infinity branches. If (13) is written as

$$K_c = \frac{b_i}{a_i} = \frac{bc_i}{ac_i} = \frac{b}{a}$$
$$\text{where} \quad i = n, n-1, \ldots, n-(m-1) \tag{17}$$

then (3) can be written as follows:

$$\Delta(s)$$
$$= 1 - K\left\{\frac{a[c_n s^n + c_{n-1}s^{n-1} + \cdots + c_{n-(m-1)}s^{n-(m-1)}]}{b[c_n s^n + c_{n-1}s^{n-1} + \cdots + c_{n-(m-1)}s^{n-(m-1)}]}\right\}$$
$$\cdot \frac{\left\{1 + \dfrac{a_{n-m}s^{n-m} + a_{n-m-1}s^{n-m-1} + \cdots + a_1 s + a_0}{a[c_n s^n + c_{n-1}s^{n-1} + \cdots + c_{n-(m-1)}s^{n-(m-1)}]}\right\}}{\left\{1 + \dfrac{b_{n-m}s^{n-m} + b_{n-m-1}s^{n-m-1} + \cdots + b_1 s + b_0}{b[c_n s^n + c_{n-1}s^{n-1} + \cdots + c_{n-(m-1)}s^{n-(m-1)}]}\right\}}$$
$$= 0 \tag{18}$$

or as shown in (19) at the bottom of the next page.

Equation (19) is reversed by dividing the numerator of the right part by its denominator. When s goes to infinity, the term s^{m-1} and the terms after it can be eliminated. Thus, (19) is approximated as

$$\left(\frac{1}{b-aK}\right)^{1/m}$$

$$=\left(\frac{c_n}{a_{n-m}K-b_{n-m}}\right)^{1/m} s$$

$$\cdot\left\{1+\frac{1}{c_n}\left[c_{n-1}-\frac{c_n(a_{n-m-1}K-b_{n-m-1})}{a_{n-m}K-b_{n-m}}\right]\frac{1}{s}\right\}^{1/m}. \tag{20}$$

By using the binomial series to approximate (20) and substituting K_c in it, the following relation will be obtained:

$$\left(\frac{1}{b-aK_c}\right)^{1/m}$$

$$=\left(\frac{c_n}{a_{n-m}K_c-b_{n-m}}\right)^{1/m}$$

$$\cdot\left\{s+\frac{1}{mc_n}\left[c_{n-1}-\frac{c_n(a_{n-m-1}K_c-b_{n-m-1})}{a_{n-m}K_c-b_{n-m}}\right]\right\}. \tag{21}$$

For going to the infinity part of the branch, K goes to K_c from the left side, so $1/(b-aK_c)$ is positive.

By replacing c_n and c_{n-1} from (17) and using the Demoivre theorem with the assumption of $s=\sigma+j\omega$, the following equation is obtained from (21):

$$\omega=\tan\frac{2i\pi}{m}\left[\sigma-\left(\frac{\frac{a_{n-m-1}K_c-b_{n-m-1}}{a_{n-m}K_c-b_{n-m}}-\frac{c_{n-1}}{c_n}}{m}\right)\right]. \tag{22}$$

For coming from the infinity part of the branch, K goes to K_c from the right side, so $1/(b-aK_c)$ is negative. Use of the Demoivre theorem results in an equation similar to (22) for coming from the infinity part of the branch as follows:

$$\omega=\tan\frac{(2i+1)\pi}{m}\left[\sigma-\left(\frac{\frac{a_{n-m-1}K_c-b_{n-m-1}}{a_{n-m}K_c-b_{n-m}}-\frac{c_{n-1}}{c_n}}{m}\right)\right]. \tag{23}$$

Equations (22) and (23) are the asymptotes equations of root locus of these systems. According to (22) and (23), when the

locus of these systems has m branches passing through infinity, there are $2m$ asymptotes that are centered at a point on the real axis with the following coordinate:

$$\sigma=\frac{\frac{a_{n-m-1}K_c-b_{n-m-1}}{a_{n-m}K_c-b_{n-m}}-\frac{a_{n-1}K_c}{b_n}}{m}. \tag{24}$$

The angles of the asymptotes of branches passing through infinity that are going to infinity with respect to the real axis are given by

$$\theta_i=\frac{2i\pi}{m}\qquad\text{where}\quad i=0,1,2,\ldots,m-1. \tag{25}$$

The angles of the asymptotes of the branches passing through infinity, which are coming from infinity with respect to the real axis, are obtained as follows:

$$\theta_i=\frac{(2i+1)\pi}{m}\qquad\text{where}\quad i=0,1,2,\ldots,m-1. \tag{26}$$

Equations (25) and (26) are similar to the corresponding equations for obtaining the angles of the asymptotes in the root locus of positive and negative feedback systems, respectively.

Now, these theorems are applied to the examples that were presented at the beginning of this paper.

For the first example, the critical gain is

$$K_c=\frac{b_4}{a_4}=\frac{3}{2}\neq\frac{b_3}{a_3}=\frac{8}{5}.$$

Therefore, there is only one branch passing through infinity and two asymptotes in the root locus of this system. The intersection point of the asymptotes with each other on the real axis is $\sigma=-2.5$. The angle of the asymptote that is going to infinity is $\theta_0=0$, and the angle of the asymptote that is coming from infinity is $\theta_0=\pi$.

For the second example, the critical gain is

$$K_c=\frac{b_4}{a_4}=\frac{3}{2}=\frac{b_3}{a_3}=\frac{6}{4}=\frac{b_2}{a_2}=\frac{9}{6}=\frac{b_1}{a_1}=\frac{12}{8}$$

$$\neq\frac{b_0}{a_0}=\frac{20}{-10}.$$

Thus, there are four branches passing through infinity and eight asymptotes in the root locus of this system. The intersection point of the asymptotes with each other on the real axis is $\sigma=-0.5$. The angles of the asymptotes that are going to infinity are $\theta_0=0$, $\theta_1=\pi/2$, $\theta_2=\pi$, $\theta_3=3\pi/2$, and the angles of the asymptotes that are coming from infinity are $\theta_0=\pi/4$, $\theta_1=3\pi/4$, $\theta_2=5\pi/4$, $\theta_3=7\pi/4$.

The plots shown in Figs. 1 and 2 can verify the above results for each example.

$$b-aK=\frac{(a_{n-m}K-b_{n-m})s^{n-m}+(a_{n-m-1}K-b_{n-m-1})s^{n-m-1}+\cdots+(a_1K-b_1)s+(a_0K-b_0)}{c_ns^n+c_{n-1}s^{n-1}+\cdots+c_{n-(m-1)}s^{n-(m-1)}}. \tag{19}$$

III. REALIZATION CONSIDERATION

Consider a system with a unity positive feedback whose closed-loop transfer function is as follows and satisfies (13):

$$T(s) = \frac{KG(s)}{1 - KG(s)}. \tag{27}$$

Substituting (2) into (27) gives (28) as shown at the bottom of the page.

$T(s)$ is a proper function and can be realized because

$$\lim_{s \to \infty} T(s) = \frac{Ka_n}{b_n - Ka_n} = \text{constant}. \tag{29}$$

When the system's gain K approaches the critical gain according to (6), $T(s)$ becomes as shown in (30) at the bottom of the page. In this case, $T(s)$ becomes an improper function, and there exists no realization for this system because $\lim_{s \to \infty} T_c(s) = \infty$.

IV. STABILITY CONSIDERATION

In this section, a theorem is presented about the stability of these systems for critical gain.

Theorem 5: A system with a loop transfer function given by (2) is unstable when the gain is on an open interval centered at the critical gain.

Proof 5: Since the right end of the real axis is always part of the positive feedback root locus, there exists an infinite point in the right side of the real axis that belongs to the root locus of these systems. Infinite points of locus are caused only by critical gain. So, this infinite pole on the right side of the real axis corresponds to critical gain. Thus, it is concluded that these types of systems are certainly unstable for critical gain.

It is interesting to point out that if the characteristic equation of the system for critical gain is calculated, it would be a polynomial with the order of at most $(n - 1)$ which may have poles all possessing negative real parts and, therefore, appears to be stable for the critical gain. But one has to make a note that the cause of system instability at the critical gain is the positive infinite pole that exists on a real axis and causes the order reduction of a characteristic equation for critical gain.

V. GAIN PLOTS

The gain plots were introduced by Kurfess and Nagurka [10] in 1991. These plots consist of a magnitude–gain plot and an angle–Gain plot. In a magnitude–gain plot, the x-axis represents
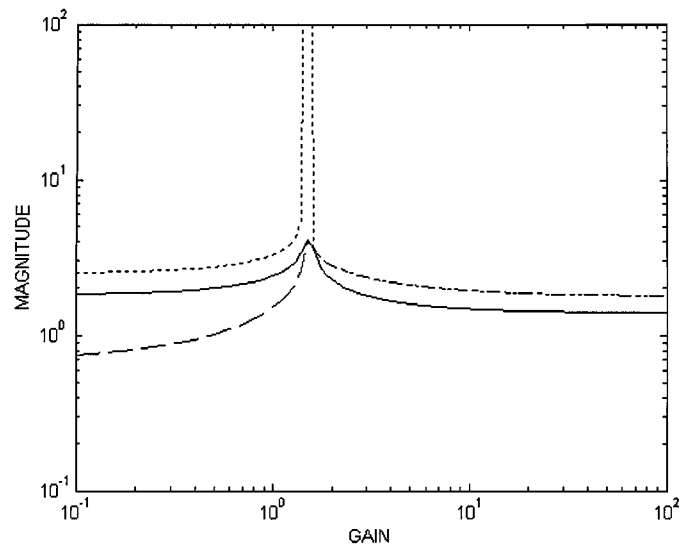

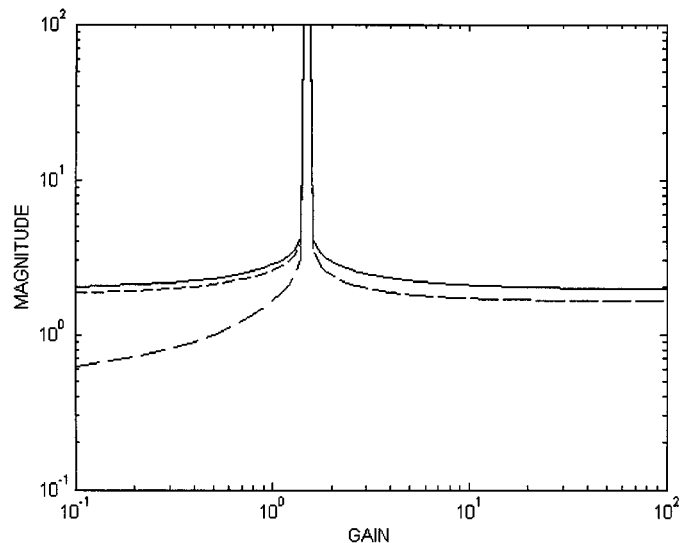Fig. 3. Magnitude–Gain plot of example 1.

Fig. 4. Magnitude–Gain plot of example 2.

gain K and the y-axis denotes the magnitude of the closed-loop poles. Whereas, in an angle–Gain plot, the x-axis shows gain and the y-axis is the angle of closed-loop poles. The relationship between gain plots and a root locus diagram is similar to the relation that exists between Bode plots and a Nyquist plot.

For systems considered in this paper, there exist m branches in a magnitude–gain plot that are going to infinity and coming

$$T(s) = \frac{Ka_n s^n + Ka_{n-1}s^{n-1} + \cdots + Ka_1 s + Ka_0}{(b_n - Ka_n)s^n + (b_{n-1} - Ka_{n-1})s^{n-1} + \cdots + (b_1 - Ka_1)s + (b_0 - Ka_0)}. \tag{28}$$

$$T_c(s) = \frac{a_n b_n s^n + a_{n-1}b_n s^{n-1} + \cdots + a_1 b_n s + a_0 b_n}{(a_n b_{n-m} - b_n a_{n-m})s^{n-m} + \cdots + (a_n b_1 - b_n a_1)s + (a_n b_0 - b_n a_0)}. \tag{30}$$
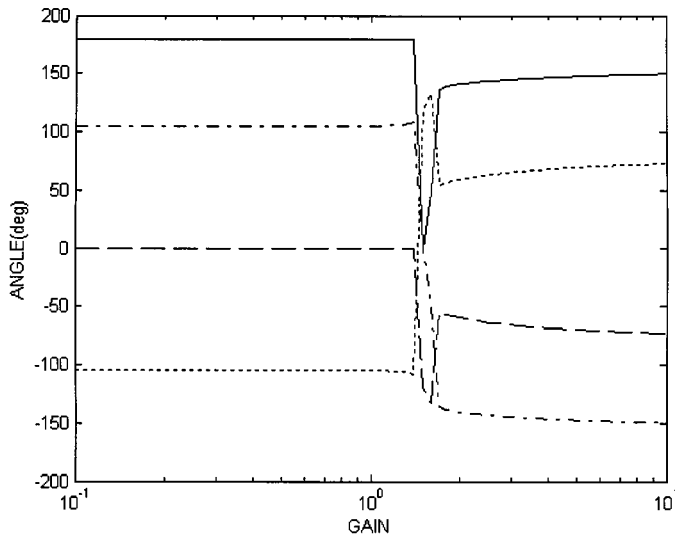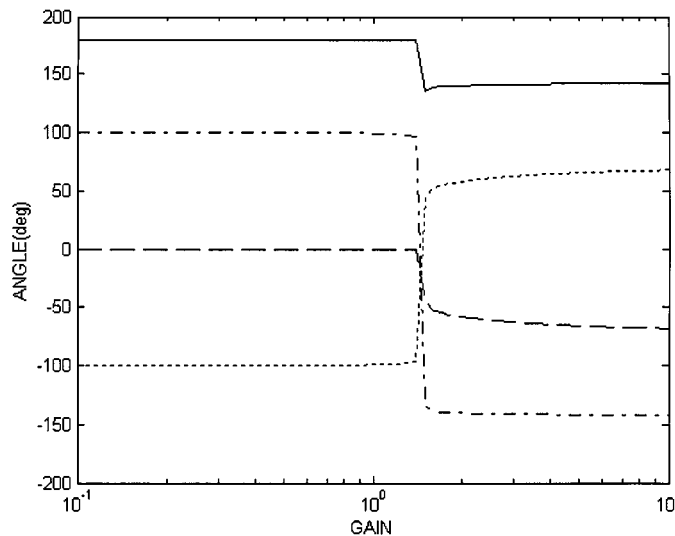
Fig. 5.   Angle–Gain plot of example 1.

Fig. 6.   Angle–Gain plot of example 2.

back from infinity at the same critical gain. The angles of asymptotes for these branches are 90°. The angle–Gain plot for these systems has m branches that approach the critical gain with the angles defined by (25), and leave critical gain with the angles defined by (26). Since each branch in the angle–Gain plot has two different values at the critical gain, there are discontinuities in the plot at the critical gain. Figs. 3 and 4 show the magnitude–gain plot, and Figs. 5 and 6 present the angle–Gain plot for the first and second examples, respectively.

VI. CONCLUSION

In this paper, we have shown some of the steps in the method of constructing complementary root locus are not valid for systems with loop transfer functions that are not strictly proper. New branches called "branches passing through infinity" have been introduced, and their properties have been demonstrated in four theorems. The investigations of the realization and stability of these systems show they are neither realizable nor stable for

critical gain. The gain plots demonstrate clear pictures of how critical gain makes these systems unstable.

The branches passing through infinity have not been previously investigated and could be presented to students through educational activities and exercises in the classroom. It is recommended that first a few examples based on the examples shown in this paper be generated to bring the attention of the students to the shortcoming of the Evans root locus method and then the formulations provided here be presented. These activities would complement the lectures on the root locus topic.

Most of the software that provide root locus plots use an algorithm which generates a table by changing the gain in the specified range and then finds the roots of the characteristic equation. But, if the value used for gain in these algorithms becomes equal to the critical gain of the system, the software would have a division by zero situation that would cause the computer to halt. Thus, the selection of any value equal to critical gain of the system should be avoided in software design.

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